

# Natural Language Reasoning with a Natural Theorem Prover

## Day 4: Natural Language Theorem Proving

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# Where are we now

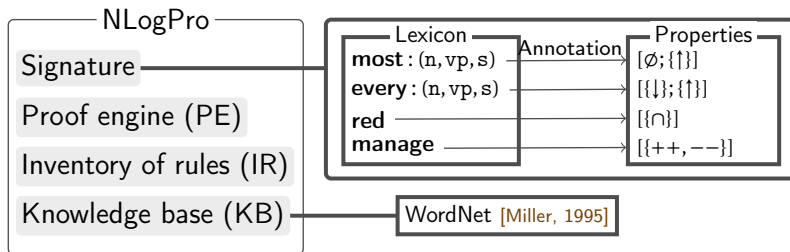
What we have done so far:

- Introduce Natural Tableau: a tableau system for natural logic, with more natural rules, with LLFs types with syntactic and semantic types
- Obtaining LLFs from CCG derivations of CCG parsers: simplifying, fixing and type-raising
- Rules that tackle erroneous PP-attachments (optional if the performance needs it)

What is today's plan:

- Describe a Natural Tableau-based theorem prover for natural language
- Describing the SICK and FraCaS NLI datasets
- Evaluation on FraCaS (on SICK will be tomorrow)
- Running the prover on google colab

# Natural logic theorem prover (NLogPro)



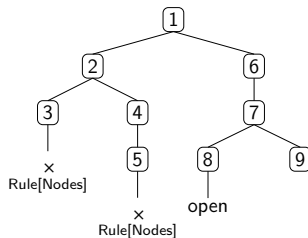
KB uses 4 relations from WordNet 3.0 ([online version](#)):

- derivation
- similarity
- hyponymy/hypernymy
- antonymy

⚠ No word sense disambiguation system is used.

# Two data structures

The proof engine builds both a tree and a list structures:



$Br_1 : \langle \text{History}_1, \text{Entities}_1 \rangle$  1—2—3

$Br_2 : \langle \text{History}_2, \text{Entities}_2 \rangle$  1—2—4—5

$Br_3 : \langle \text{History}_3, \text{Entities}_3 \rangle$  1—6—7—8

$Br_4 : \langle \text{History}_4, \text{Entities}_4 \rangle$  1—6—7—9

# Some derivable rules

Derivable rules are shortcuts for several rule applications.

$$\exists_F^n$$

$q_{n,vp,s}NV : [] : \mathbb{F}$ $N : [c_e] : \mathbb{T}$
$V : [c] : \mathbb{F}$
$q \in \{\mathbf{a, some, the, s}\}$

$$\forall_T^n$$

$q_{n,vp,s}NV : [] : \mathbb{T}$ $N : [c_e] : \mathbb{T}$
$V : [c] : \mathbb{T}$
$q \in \{\mathbf{every, the}\}$

$$\text{NO}_T^n$$

$\mathbf{no}_{n,vp,s}NV : [] : \mathbb{T}$ $N : [c_e] : \mathbb{T}$
$V : [c] : \mathbb{F}$

$$\exists_F^\nu$$

$q_{n,vp,s}NV : [] : \mathbb{F}$ $V : [c_e] : \mathbb{T}$
$N : [c] : \mathbb{F}$
$q \in \{\mathbf{a, some, the, s}\}$

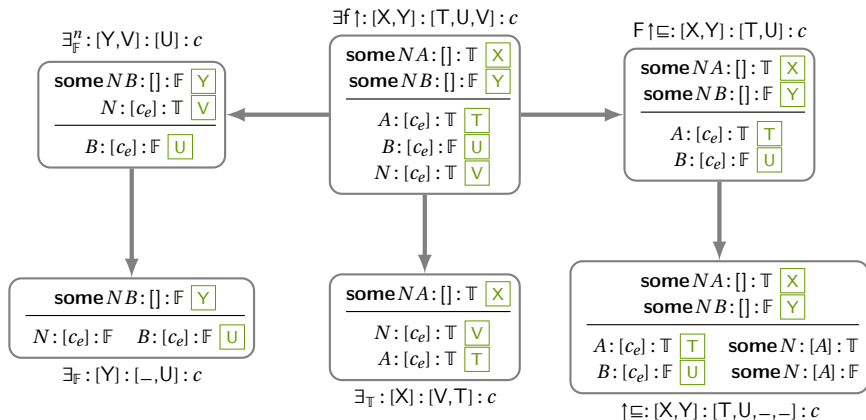
$$\forall_T^\nu$$

$q_{n,vp,s}NV : [] : \mathbb{T}$ $V : [c_e] : \mathbb{F}$
$N : [c] : \mathbb{F}$
$q \in \{\mathbf{every, the}\}$

$$\text{NO}_T^\nu$$

$\mathbf{no}_{n,vp,s}NV : [] : \mathbb{T}$ $V : [c_e] : \mathbb{T}$
$N : [c] : \mathbb{F}$

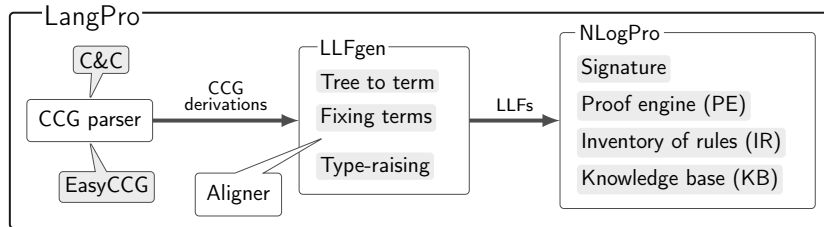
# Rule application subsumption



$$\exists_F^n : [Y, V] : [U] : c \Rightarrow \exists_F : [Y] : [-, U] : c$$

# Natural language theorem prover (LangPro)

Chaining a CCG parser, the LLF generator and NLogPro results in a theorem prover for natural language.



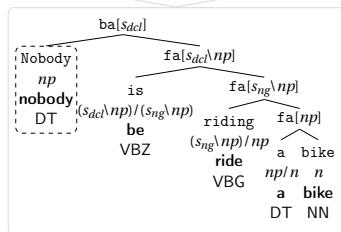
Online demo: <http://naturallogic.pro/LangPro>

GitHub repo: <https://github.com/kovvalsky/LangPro>

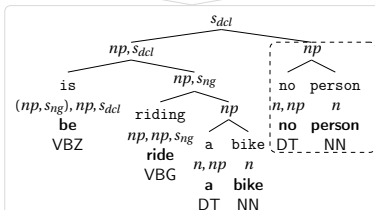
# LangPro in action

SICK-2865: Nobody is riding a bike  $\Rightarrow$  Two people are riding a bike

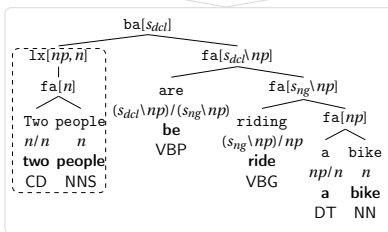
the C&C parser



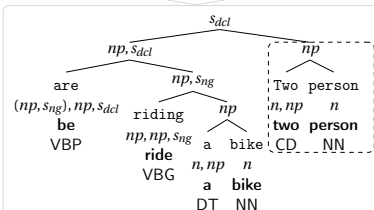
Fixing



the C&C parser

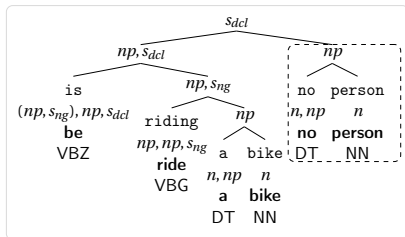


Fixing



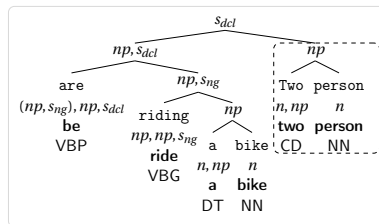


# LangPro in action (2)



Type-raising

no person (be ( $\lambda x$ . (a bike) ( $\lambda y$ . ride y x)))  
 a bike ( $\lambda x$ . no person (be (ride x)))



Type-raising

two person (be ( $\lambda x$ . (a bike) ( $\lambda y$ . ride y x)))  
 a bike ( $\lambda x$ . two person (be (ride x)))

Proving by PE using IR &amp; KB

intial nodes for entailment checking:  
 no person (be ( $\lambda x$ . (a bike) ( $\lambda y$ . ride y x))): []:  $\top$   
 two person (be ( $\lambda x$ . (a bike) ( $\lambda y$ . ride y x))): []:  $\top$

intial nodes for contradiction checking:  
 no person (be ( $\lambda x$ . (a bike) ( $\lambda y$ . ride y x))): []:  $\top$   
 two person (be ( $\lambda x$ . (a bike) ( $\lambda y$ . ride y x))): []:  $\top$

# LangPro in action (3)

1 no person (be( $\lambda x$ . (a bike) ( $\lambda y$ . ride  $y$   $x$ ))): [] :  $\mathbb{T}$

2 two person (be ( $\lambda x$ . (a bike) ( $\lambda y$ . ride  $y$   $x$ ))): [] :  $\mathbb{T}$

$\exists_{\mathbb{T}}[2]$  |

3 person: [ $c$ ] :  $\mathbb{T}$

4 be( $\lambda x$ . (a bike) ( $\lambda y$ . ride  $y$   $x$ )): [ $c$ ] :  $\mathbb{T}$

$\text{no}_{\mathbb{T}}^n[1,4]$  |

5 person: [ $c$ ] :  $\mathbb{F}$

6  $\times$

$$\frac{\text{no } A \ B : [] : \mathbb{T} \quad A : [c] : \mathbb{T}}{B : [c] : \mathbb{F}} \text{no}_{\mathbb{T}}^n$$

$$\frac{N^{\text{CD}} \ A \ B : [] : \mathbb{T}}{A : [c] : \mathbb{T} \quad B : [c] : \mathbb{T}} \exists_{\mathbb{T}}$$

# The SICK dataset

SICK [Marelli et al., 2014b] contains Sentences Involving Compositional Knowledge:

- 10K Text-Hypothesis pairs generated semi-automatically and annotated by humans with three labels: E, C, & N.
- Contains no encyclopedic knowledge, no named entities, relatively small vocabulary, less multiword expressions and no lengthy sentences ( $\approx 9$  words per sentence).
- Contradictions (86%) rely too much on negative words and antonyms [Lai and Hockenmaier, 2014].
- A benchmark for the SemEval-14 RTE task [Marelli et al., 2014a]: Trial (5%), Train (45%), and test (50%).
- 84% of crowd workers' labels match the majority, i.e, gold labels.

# SICK construction

Original pair			
S0a: <i>A sea turtle is hunting for fish</i>	S0b: <i>The turtle followed the fish</i>		
Normalized pair			
S1a: <i>A sea turtle is hunting for fish</i>	S1b: <i>The turtle is following the fish</i>		
Expanded pair			
Similar meaning			
S2a: <i>A sea turtle is hunting for food</i>	S2b: <i>The turtle is following the red fish</i>		
Logically contradictory or at least highly contrasting meaning			
S3a: <i>A sea turtle is not hunting for fish</i>	S3b: <i>The turtle isn't following the fish</i>		
Lexically similar but different meaning			
S4a: <i>A fish is hunting for a turtle in the sea</i>	S4b: <i>The fish is following the turtle</i>		
Normalized sentence pairs		Score	Label
S1a: <i>A sea turtle is hunting for fish</i>	S2a: <i>A sea turtle is hunting for food</i>	4.5	E
S3a: <i>A sea turtle is not hunting for fish</i>	S1a: <i>A sea turtle is hunting for fish</i>	3.4	C
S4a: <i>A fish is hunting for a turtle in the sea</i>	S1a: <i>A sea turtle is hunting for fish</i>	3.9	N
S2b: <i>The turtle is following the red fish</i>	S1b: <i>The turtle is following the fish</i>	4.6	E
S1b: <i>The turtle is following the fish</i>	S3b: <i>The turtle isn't following the fish</i>	4	C
S1b: <i>The turtle is following the fish</i>	S4b: <i>The fish is following the turtle</i>	3.8	C
S1a: <i>A sea turtle is hunting for fish</i>	S2b: <i>The turtle is following the red fish</i>	4	N
S1a: <i>A sea turtle is hunting for fish</i>	S3b: <i>The turtle isn't following the fish</i>	3.2	N
S4b: <i>The fish is following the turtle</i>	S1a: <i>A sea turtle is hunting for fish</i>	3.2	N
S1b: <i>The turtle is following the fish</i>	S2a: <i>A sea turtle is hunting for food</i>	3.9	N
S1b: <i>The turtle is following the fish</i>	S3a: <i>A sea turtle is not hunting for fish</i>	3.4	N
S4a: <i>A fish is hunting for a turtle in the sea</i>	S1b: <i>The turtle is following the fish</i>	3.5	N
S1a: <i>A sea turtle is hunting for fish</i>	S1b: <i>The turtle is following the fish</i>	3.8	N

# SICK examples and stats

SICK-1241 GOLD: neutral

A woman is dancing and singing with other women

A woman is dancing and singing in the rain

SICK-341 GOLD: contradiction

There is no girl with a black bag on a crowded train

A girl with a black bag is on a crowded train

SICK-8381 GOLD: entailment

The young girl in blue is having fun on a slide

The young girl in blue is enjoying a slide

Relatedness	neutral	contradiction	entailment	Total
[1,2) range	10%	0%	0%	10% (923)
[2,3) range	13%	1%	0%	14% (1373)
[3,4) range	28%	10%	1%	29% (3872)
[4,5] range	7%	3%	27%	37% (3672)
Total	56.86% (5595)	14.47% (1424)	28.67% (2821)	9840

# The FraCaS dataset

The FraCaS test suite [Cooper et al., 1996] was an early attempt to creating a semantic benchmark for NLP systems.

- Contains 346 problems, 45% of which are multi-premised.
- Covers GQs, plurals, anaphora, ellipsis, adjectives, comparatives, temporal reference, verbs and attitudes.
- Three-way annotated by the authors of the dataset.
- Contains some ambiguous sentences and a few erroneous problems.
- Requires almost no lexical or world knowledge

Later, the FraCaS question-answer pairs were converted into an NLI format [MacCartney and Manning, 2007]: [online version](#)

# FraCaS NLI problems

FraCaS-26    GOLD: entailment

Most Europeans are resident in Europe

All Europeans are people

All people who are resident in Europe can travel freely within Europe

---

Most Europeans can travel freely within Europe

FraCaS-61    GOLD: undefined

Both female commissioners used to be in business.

---

Both commissioners used to be in business.

FraCaS-171    GOLD: entailment

John wants to know how many men work part time.

And women.

---

John wants to know how many women work part time.

FraCaS-87    GOLD: entailment

Every representative and client was at the meeting.

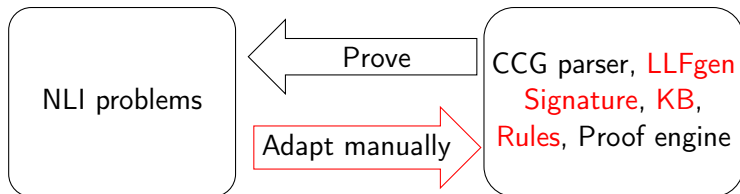
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Every representative was at the meeting.

# Learning phase

The prover LangPro is (semi-automatically) trained on the NLI datasets [Abzianidze, 2016a].

- **Adaptation:**



Used datasets: SICK-trial and FraCaS

- **Development:**

Finding optimal values for certain parameters of the prover based on its performance on SICK-train.

**NB:** Only C&C parser is used in the learning phase in order to test LangPro for an unseen parser, EasyCCG, later.



# Adaptation: negative cases

We avoid fitting to the data and adopting unsound and non-general solutions.

The problems that were not solved during the adaptation:

- Sentence is not recognised as of category *S* or failed to be parsed
- The error in analysis is too specific to fix:

At most ten commissioners spend time at home  
*(S/S)/NP N/N N/N N (VP/PP)/NP N PP/NP N*

- Lexical relation is context dependent:

SICK-4505 GOLD: entailment

The doctors are healing a **man**

The doctor is helping the **patient**

SICK-384 GOLD: entailment

A white and tan dog is running through the **tall and green grass**

A white and tan dog is running through a **field**

# Adaptation: positive cases

The problems that were solved by upgrading one of the components of the prover:

- Treat **few** as  $\downarrow$  in its 1st arg (*absolute* reading):

FraCaS-76

GOLD: entailment

Few committee members are from southern Europe

Few female committee members are from southern Europe

- Introduce **fit**  $\sqsubseteq$  **apply** and **food**  $\sqsubseteq$  **meal**:

SICK-4734

GOLD: entailment

A man is **fitting** a silencer to a pistol

A man is **applying** a silencer to a gun

SICK-5110

GOLD: entailment

A chef is preparing some **food**

A chef is preparing a **meal**

# Development phase

Optimal values of the following parameters are searched:

- The number of word senses to consider at the same time;
- The upper bound for the number of rule applications;
- Whether to use a term aligner:

- **Weak aligner** aligns everything except terms of type np:

SICK-1022 GOLD: contradiction

A woman is **wearing sunglasses of large size** and **is holding newspapers in both hands**

There is no woman **wearing sunglasses of large size** and **holding newspapers in both hands**

SICK-727 GOLD: contradiction

The **man in a grey t-shirt is sitting on a rock in front of the waterfall**

There is no **man in a grey t-shirt sitting on a rock in front of the waterfall**

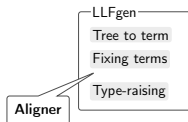
- **Strong aligner** aligns everything except terms of type np with ↓arg.

SICK-423 GOLD: contradiction

**Two men** are not **holding fishing poles**

**Two men** are **holding fishing poles**

- Efficiency criterion of tableau rules.



# Efficiency criterion

Tableau rules have the following properties:

- Non-branching or branching (so called,  $\alpha$  or  $\beta$  rules);
- Semantic equivalence vs proper entailment;
- Consuming (so called,  $\gamma$  rule) vs non-consuming;
- Producing (so called,  $\delta$  rule) vs non-producing.

An example of an efficiency criterion:

$$EC = \langle \text{nonBr}, \text{semEqui}, \text{nonConsum}, \text{nonProd} \rangle$$

An efficiency vectors based on the  $EC$  efficiency criterion:

- $V_{EC}(\wedge_{\top}) = 1111$
- $V_{EC}(\vee_{\top}) = 0111$
- $V_{EC}(\exists_{\top}) = 1110$
- $V_{EC}(\exists_{\text{F}}) = 0001$

**What is the optimal efficiency criterion?**

# Greedy search for optimal parameters

Acc%	Prec%	Rec%	Sense	Efficiency criterion	Aligner	RAL	Parser
75.09	98.5	43.6	1	[nonP,nonB,equi,nonC]	No	200	C&C
76.42	98.3	46.8	1-5	-	-	-	-
76.89	97.8	48.1	All	-	-	-	-
78.44	97.9	51.7	-	[equi,nonB,nonP,nonC]	-	-	-
79.33	97.9	53.8	-	-	Weak	-	-
81.5	97.7	59.0	-	-	Strong	-	-
81.53	97.7	59.1	-	-	Strong	400	-
81.38	98.0	58.5	-	-	Strong	400	EasyCCG
82.6	97.7	61.6	-	-	Strong	400	Both

The results are given on the SICK-train problems.

FraCaS-21 GOLD: entailment

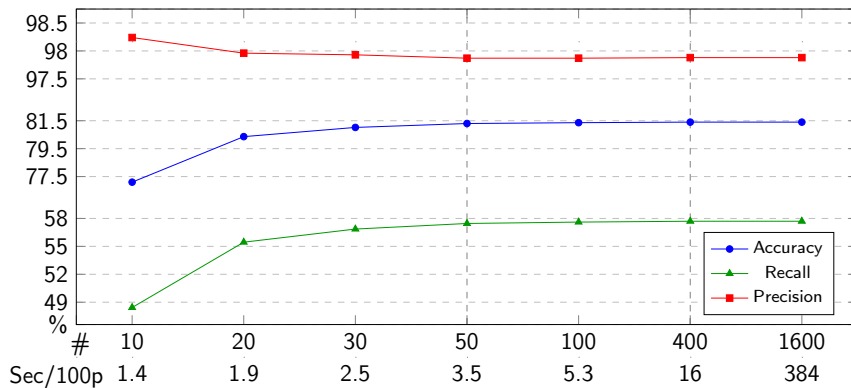
The residents of member states have the right to live in Europe

All residents of member states are individuals

Every individual who has the right to live in Europe can travel freely within Europe

The residents of member states can travel freely within Europe

# Efficient and optimal rule application numbers



The results are given on the SICK-train problems.

# Solving FraCaS [Abzianidze, 2016b]

LangPro with C&C					LangPro with EasyCCG				
Gold\ccLP	yes	no	unk		Gold\easyLP	yes	no	unk	
yes	<b>51</b>	0	19 + 4	+	yes	<b>52</b>	0	22	=
no	1	<b>14</b>	2		no	1	<b>12</b>	4	
unk	1	0	<b>44 + 6</b>		unk	2	0	<b>49</b>	
P = .97, R = .71, Acc = .81					P = .96, R = .70, Acc = .80				

LangPro				
Gold\LP	yes	no	unk	=
yes	<b>60</b>	0	14	
no	1	<b>14</b>	2	
unk	2	0	<b>49</b>	
P = .96, R = .81, Acc = .87				

FraCaS-109 GOLD: contradiction LP: entailment

Just one accountant attended the meeting

Some accountants attended the meeting

## Related work (FraCaS)

[MacCartney and Manning, 2008] and [Angeli and Manning, 2014] employ a natural logic that is driven by sentence edits.

[Lewis and Steedman, 2013] employ Boxer-style [Bos et al., 2004] translation into FOL, Prover9 and distributional relation clustering.

[Mineshima et al., 2015, Haruta et al., 2020] also uses the Boxer-style translation but some HOGQs are treated as higher-order terms. Their inference system is implemented in the proof assistant Coq.

[Tian et al., 2014] and [Dong et al., 2014] uses abstract denotations obtained from DCS trees [Liang et al., 2011]:  $\text{man} \subset \pi_{\text{subj}}(\text{read} \cap (W_{\text{subj}} \times \text{book}_{\text{obj}}))$

[Bernardy and Chatzikyriakidis, 2017] uses Grammatical Framework and Coq.

[Hu et al., 2019] monotonicity calculus with trees obtain from CCG parsers.

[Kim et al., 2021] monotonicity reasoning with Unscoped Episodic Logical Forms.



# Comparison on FraCaS


Sec (Sing/All)	Single-premised (Acc %)									Overall (Acc %)								
	BL	NL07,08	LS	NL14	T14a,b	M15	K21	LP	BL	LS13	T14a,b	M15	H20	HM19	BC21 <sup>G</sup>	K21	LP	
1 GQs (44/74)	45	84 98	70	95	80 93	82	73	93	50	62	80 95	78	97	88	93	70	95	
2 Plur (24/33)	58	42 75	-	38	-	67	-	75	61	-	-	67	-	-	79	-	73	
5 Adj (15/22)	40	60 80	-	87	-	87	-	87	41	-	-	68	82	-	86	-	77	
9 Att (9/13)	67	56 89	-	22	-	78	-	100	62	-	-	77	92	-	85	-	92	
1,2,5,9 (92/142)	50	- 88	-	-	-	78	-	88	52	-	-	74	-	-	88	-	87	

BL majority baseline, NL07 [MacCartney and Manning, 2007], NL08 [MacCartney and Manning, 2008], NL14 [Angeli and Manning, 2014], LS13 [Lewis and Steedman, 2013], M15 [Mineshima et al., 2015], T14a [Tian et al., 2014], T14b [Dong et al., 2014], HM19 [Hu et al., 2019], H20 [Haruta et al., 2020], K21 [Kim et al., 2021], and BC21 [Bernardy and Chatzikyriakidis, 2021] (with gold trees)







# Conclusion

- The theorem prover for natural logic;
- The theorem prover for natural language is a pipeline:  
CCG parser + LLFgen + natural logic prover + WordNet;
- Play with it: <http://naturallogic.pro>
- Clone or fork it: <https://github.com/kovvalsky/LangPro>








# References I

-  Abzianidze, L. (2016a). *A natural proof system for natural language*. PhD thesis, Tilburg University.
-  Abzianidze, L. (2016b). Natural solution to fracas entailment problems. In *Proceedings of the Fifth Joint Conference on Lexical and Computational Semantics*, pages 64–74, Berlin, Germany. Association for Computational Linguistics.
-  Angeli, G. and Manning, C. D. (2014). Naturalli: Natural logic inference for common sense reasoning. In *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP)*.
-  Bernardy, J.-P. and Chatzikyriakidis, S. (2017). A type-theoretical system for the FraCaS test suite: Grammatical framework meets coq. In *IWCS 2017 - 12th International Conference on Computational Semantics - Long papers*.
-  Bernardy, J.-P. and Chatzikyriakidis, S. (2021). Applied temporal analysis: A complete run of the FraCaS test suite. In *Proceedings of the 14th International Conference on Computational Semantics (IWCS)*, pages 11–20, Groningen, The Netherlands (online). Association for Computational Linguistics.
-  Bos, J., Clark, S., Steedman, M., Curran, J. R., and Hockenmaier, J. (2004). Wide-coverage semantic representations from a ccg parser. In *Proceedings of the 20th International Conference on Computational Linguistics (COLING '04)*, pages 1240–1246, Geneva, Switzerland.
-  Cooper, R., Crouch, D., Eijck, J. V., Fox, C., Genabith, J. V., Jaspars, J., Kamp, H., Milward, D., Pinkal, M., Poesio, M., Pulman, S., Briscoe, T., Maier, H., and Konrad, K. (1996). *FraCaS: A Framework for Computational Semantics*. Deliverable D16.

# References II

- 
- Dong, Y., Tian, R., and Miyao, Y. (2014). Encoding generalized quantifiers in dependency-based compositional semantics. In *Proceedings of the 28th Pacific Asia Conference on Language, Information, and Computation*, pages 585–594, Phuket, Thailand. Department of Linguistics, Chulalongkorn University.
- 
- Haruta, I., Mineshima, K., and Bekki, D. (2020). Combining event semantics and degree semantics for natural language inference. In *Proceedings of the 28th International Conference on Computational Linguistics*, pages 1758–1764, Barcelona, Spain (Online). International Committee on Computational Linguistics.
- 
- Hu, H., Chen, Q., and Moss, L. (2019). Natural language inference with monotonicity. In *Proceedings of the 13th International Conference on Computational Semantics - Short Papers*, pages 8–15, Gothenburg, Sweden. Association for Computational Linguistics.
- 
- Kim, G., Juvekar, M., Ekmekciu, J., Duong, V., and Schubert, L. (2021). A (mostly) symbolic system for monotonic inference with unscoped episodic logical forms. In *Proceedings of the 1st and 2nd Workshops on Natural Logic Meets Machine Learning (NALOMA)*, pages 71–80, Groningen, the Netherlands (online). Association for Computational Linguistics.
- 
- Lai, A. and Hockenmaier, J. (2014). Illinois-lh: A denotational and distributional approach to semantics. In *Proceedings of the 8th International Workshop on Semantic Evaluation (SemEval 2014)*, pages 329–334, Dublin, Ireland. Association for Computational Linguistics and Dublin City University.
- 
- Lewis, M. and Steedman, M. (2013). Combined distributional and logical semantics. *Transactions of the Association for Computational Linguistics (TACL)*, 1:179–192.

# References III

- 
- Liang, P., Jordan, M. I., and Klein, D. (2011). Learning dependency-based compositional semantics. In *Association for Computational Linguistics (ACL)*, pages 590–599.
- 
- MacCartney, B. and Manning, C. D. (2007). Natural logic for textual inference. In *Proceedings of the ACL-PASCAL Workshop on Textual Entailment and Paraphrasing, RTE '07*, pages 193–200, Stroudsburg, PA, USA. Association for Computational Linguistics.
- 
- MacCartney, B. and Manning, C. D. (2008). Modeling semantic containment and exclusion in natural language inference. In Scott, D. and Uszkoreit, H., editors, *COLING*, pages 521–528.
- 
- Marelli, M., Menini, S., Baroni, M., Bentivogli, L., Bernardi, R., and Zamparelli, R. (2014a). Semeval-2014 task 1: Evaluation of compositional distributional semantic models on full sentences through semantic relatedness and textual entailment. In *Proceedings of SemEval 2014 (International Workshop on Semantic Evaluation)*, pages 1–8, East Stroudsburg PA. ACL.
- 
- Marelli, M., Menini, S., Baroni, M., Bentivogli, L., Bernardi, R., and Zamparelli, R. (2014b). A sick cure for the evaluation of compositional distributional semantic models. In Calzolari, N., Choukri, K., Declerck, T., Loftsson, H., Maegaard, B., Mariani, J., Moreno, A., Odijk, J., and Piperidis, S., editors, *Proceedings of the Ninth International Conference on Language Resources and Evaluation (LREC'14)*, Reykjavik, Iceland. European Language Resources Association (ELRA).
- 
- Miller, G. A. (1995). Wordnet: A lexical database for english. *Communications of the ACM*, 38(11):39–41.
- 
- Mineshima, K., Martínez-Gómez, P., Miyao, Y., and Bekki, D. (2015). Higher-order logical inference with compositional semantics. In *Proceedings of the 2015 Conference on Empirical Methods in Natural Language Processing*, pages 2055–2061, Lisbon, Portugal. Association for Computational Linguistics.

# References IV



Tian, R., Miyao, Y., and Matsuzaki, T. (2014). Logical inference on dependency-based compositional semantics. In *Proceedings of the 52nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 79–89, Baltimore, Maryland. Association for Computational Linguistics.